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6. AUTHOR(S)
Bullo, Francesco

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Coordinated Science Laboratory
University of Illinois at Urbana–Champaign
1308 W. Main St, Urbana, IL 61801

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13. ABSTRACT (Maximum 200 words)

The objective of this research has been the design and validation of innovative methods for motion planning of highly mobile all-terrain vehicles and motion coordination for multi-vehicle networks.

The initial focus of this research was on algorithms for (single) vehicle trajectory generation. We have studied fast local trajectory optimization algorithms for vehicles with limited control authority.

The main research focus has been on a novel class of asynchronous distributed coordination algorithms for multi-vehicles networks. We have developed coverage and coordination algorithms for communication-constrained vehicle models. The algorithms are spatially distributed in the sense that only spatially localized information is required for their implementation. We believe this is a fundamental improvement over the current state-of-the-art in motion coordination.s

Finally, we have developed an outdoor experimental platform, called the "Multi-Rover Network Laboratory," consisting of eight model rovers equipped with embedded computers, wireless interfaces, and sensors.

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"Trajectories for Locomotion Systems: A Geometric and Computational Approach via Series Expansions"
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Francesco Bullo
Coordinated Science Laboratory, University of Illinois at Urbana-Champaign
1308 W. Main St, Urbana, IL 61801, United States
Tel: (217) 333-0656, Fax: (217) 244-1653,
Email: bullo@uiuc.edu, Url: http://motion.csl.uiuc.edu

Current Address:
Mechanical & Environmental Engineering, University of California at Santa Barbara
2338 Engineering Bldg II, Santa Barbara, CA 93106-5070
Tel: (805) 893-5169, Fax: (805) 893-8651
Email: bullo@engineering.ucsb.edu, Url: http://www.me.ucsb.edu/bullo

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Foreword

The initial focus of this research was on motion planning problems for autonomous agile vehicles. At the the project onset, careful discussions between the Dr Hua Wang, responsible Program Office, and the PI led to a shift in emphasis to the study of motion coordination problems for multi-vehicles networks.

Contents

1 Scientific Progress and Accomplishments: Motion Planning for Autonomous Vehicles

This section documents the work in [1], [2] on the design of low complexity algorithms for local motion planning problems. We have developed a methodology based on series expansions and constructive controllability for nonlinear control systems.

To exemplify our approach, let us start by briefly considering linear control systems in the usual form $\dot{x} = Ax + Bu(t)$, $x \in \mathbb{R}^n, u \in \mathbb{R}^m$. We state two established sets of results. First, the forced evolution from $x(0) = 0$ is $x(t) = \int_0^t e^{A(t-s)} Bu(s) ds$. Second, if and only if the system is controllable, the controllability Grammian is positive definite:

$$W = \int_0^T e^{A(T-s)} BB' e^{A'(T-s)} ds > 0.$$

Relying on these two facts, one can show that an (energy optimal) open-loop control to reach the target final state x_{goal} is given by

$$u(t) = B' e^{A'(T-t)} W^{-1} x_{\text{goal}}.$$

The references present a methodology to replicate these analysis and design steps for nonlinear control systems:

Step 1 - evolution via series expansions We consider a class of polynomial nonlinearities common to robotic systems and vehicles. In [1], we show how the initial value problem

$$\dot{x} = Ax + f^{[2]}(x, x) + Bu\,, \quad x(0) = 0,$$

where $f^{[2]} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a symmetric tensor, admits the solution $x(t) = \sum_{k=1}^{+\infty} x_k(t)$ with

$$x_1(t) = \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau\,, \quad \text{and} \qquad x_k(t) = \sum_{a=1}^{k-1} \int_0^t e^{A(t-\tau)} f^{[2]}(x_a(\tau), x_{k-a}(\tau)) d\tau\,.$$

This result and the more general treatment in 2(a) extends available works on Volterra series.

Step 2 - discretization methods Next, select base functions $\{\phi_j(t), j \in \{1, \ldots, n\}\}$, e.g., sinusoids or splines, and write $u(t) = \sum_j \phi_j(t) c_j$. The evolution of the nonlinear control system is a function of $c \in \mathbb{R}^n$, that can be developed in a Taylor expansion

$$x(T) = \Phi(c) = \Phi_1 c + \sum_{k=2}^{+\infty} \Phi_k(c, \ldots, c), \tag{1}$$

where Φ_1 is a matrix and $\{\Phi_k\}$ are vector-valued tensors. To solve a point-to-point motion planning problem we now need to solve this equation for $x(T) = x_{\text{goal}}$. In reference [2], we illustrate how minimum energy control problems can also be discretized and transcribed into equations similar to (1).

Step 3 - inversion via series expansions Given equation (1), a solution c_{goal} to $\Phi(c_{\text{goal}}) = x_{\text{goal}}$ exists under linear controllability assumptions, and can be written in closed form as:

$$c_{\text{goal}} = \sum_{k=1}^{+\infty} c_k\,, \quad \text{where} \qquad c_1 = \Phi_1^{-1} x_{\text{goal}}, \;\; c_k = -\Phi_1^{-1} \sum_{\substack{i_1+\cdots+i_m=k \\ i_1,\ldots,i_m<k}} \Phi_m\big(c_{i_1}, \ldots, c_{i_m}\big).$$

In reference [2] we discuss truncation methods, design an alternative iterative algorithm, and provide closed form bounds for both algorithms.

The work in [1] and [2] provides a viable methodology to design local planning algorithms. Numerical simulations on a 1-dimensional and 6-dimensional vehicle model are presented in [2]. These two papers are directly related to the deliverables outlined in the proposal narrative.

This grant also supported our research into motion planning algorithms via other algorithmic approaches. The work in [3] illustrates how local planning methods can be integrated into global planners. The work in [4] is a contribution to motion planning problems with moving obstacles. The work in [5, 6] illustrates how the proposed approach is applicable to the setting of mechanical control systems, such as manipulators and multi-body systems.

2 Scientific Progress and Accomplishments: Deployment and Coverage Control for Multi-Vehicle Networks

The objective of this research is to develop a complete set of primitives for deployment and motion coordination in multi-vehicle networks, including coverage, formation control, flocking, swarming, and rendezvous problems.

We investigate multi-vehicle coordination in a comprehensive fashion, developing fundamental modeling tools (what are appropriate motion, communication and energy consumption models?), metrics for performance analysis (when is a configuration or a coordinated move optimal?), and algorithmic design. In particular, it is of central importance to design algorithms that will gently scale with the number of vehicles and devices present in the network.

We tackle optimal deployment and coverage problems in their numerous variations. This class of problem is very broad and the features of specific formulations vary drastically with the underlying physical assumptions. Critical parameters include:

1. the environment of interest can be two or three dimensional, known or unknown, uniform or nonuniform (e.g., portions of the environment might be of greater interest), stationary or non-stationary (e.g., boundaries and nonuniformity may depend on time);
2. the deployment objectives can vary depending on the ultimate network objective: examples include search and exploration, target detection, localization and tracking, wireless communication coverage, environmental monitoring;
3. the communication and sensing characteristics of individual vehicle can be uniform or heterogeneous (e.g., antennas and sensors can be directional or omni-directional), the vehicle mobility and dynamics can vary drastically.

Our work in this area, supported by this grant, is documented in the conference submissions [7, 8, 9, 10, 11] and the journal manuscripts [12, 13, 14]. We describe some example results in the next pages.

State-of-the-art in deplyoment and motion coordination primitives

This research relies on a growing research effort focused on motion coordination algorithms for multi-vehicle networks. Key problems involve flocking [15, 16], foraging [17], rendezvous [18, 19], coverage [20], cooperative search [21], and formation control [22, 23]. It is important to note that heuristic approaches to the design of basic interaction rules have been investigated within the literature on behavior-based robotics; see [24, 25, 26, 27]. However, only recently has there been a systematic effort to design truly scalable and provably optimal algorithms; e.g., see [15, 18]. Along this line of research, no comprehensive results are currently available on how to design motion coordination primitives, ensure their correctness, and guarantee their optimality with respect to an aggregate objective.

Coverage notion and related optimization problems in (static) wireless sensor networks are discussed for example in [28, 29]. Coverage is naturally cast as a spatial resource allocation problem and studied in a discipline called location optimization [30, 31]. A final component in our approach is our intent to identify and exploit the computational-geometric and graph-theoretical structure inherent to these problems; see [32, 33, 34].

Technical approach to deployment and motion coordination

In what follows we illustrate the results we have obtained in some aspects of this broad theme. The following performance metrics and coordination algorithms are meant to illustrate the proposed approach and not to restrict our research objectives to any specific setting. The general "bottom-up" approach is to design basic behaviors, formalize the resulting network model through nonlinear and hybrid systems theory, and prove converge correctness via Lyapunov and invariant theory.

We discuss mainly deployment problems. The following material relies on and is an extensions of our recent journal submissions [12, 13, 14] (and corresponding conference submissions). Here, let Q be a region in $\mathbb{R}^3$ and let $\|\cdot\|$ be the Euclidean distance. Let $P = (p_1, \dots, p_n)$ be the location of n agents, each moving in the environment Q.

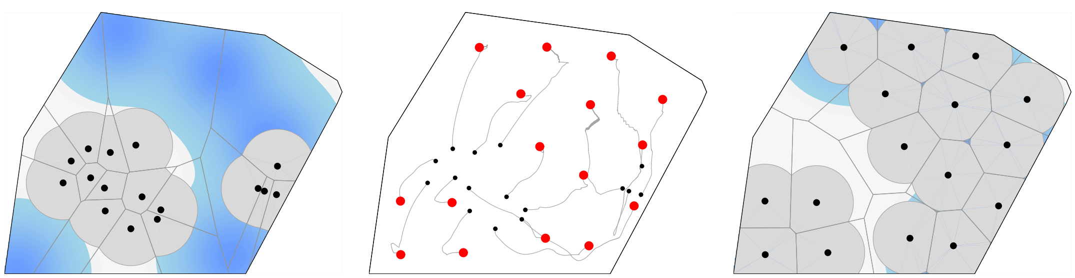
P1) Area-Coverage Deployment *[Problem setup]* Let $\phi : Q \to \mathbb{R}_+$ play the role of a distribution density function; i.e., ϕ measures how many users of the communication channel are present, or how important it is to cover a certain region in the environment Q. Because of noise and loss of resolution, the sensing or communication performance at point $q \in Q$ taken from ith agent at the position p_i degrades with the distance $\|q-p_i\|$; we describe this degradation with a monotone (decreasing) function $f : \mathbb{R}_+ \to \mathbb{R}_+$. In other words, $f(\|q-p_i\|)$ is a point-wise quantitative assessment of how poor the sensing/communication performance is. Since typical agents have limited-range footprint, it is realistic to assume that $f(\|q-p_i\|)$ is constant (equally poor) outside the sphere $B_r(p_i)$ centered at p_i of radius r. As specific example, we let $f(\|q-p_i\|)$ equals 1 if q is inside the sphere $B_r(p_i)$ and 0 otherwise. This performance function leads to the following interpretation: the agent i provides equally good sensing/communication coverage over all points in its sphere of influence.

[Performance metric] In a first approximation, let us assume that each individual agent is uniquely responsible for wireless coverage and measurements taken over a region to be determined. Let $\mathcal{W} = \{W_1, \ldots, W_n\}$ be a collection of n regions with disjoint interiors whose union is Q; we call $\mathcal{W}$ a partition of Q and W_i the dominance region of agent i. Consider the coverage performance metric $\mathcal{H}(P, \mathcal{W}) = \sum_{i=1}^{n} \int_{W_i} f(\|q-p_i\|) d\phi(q)$. The function $\mathcal{H}$ is to be maximized with respect to the agents location P and to the assignment of the dominance regions $\mathcal{W}$. One can easily see that, at fixed locations $(p_1, \ldots, p_n)$, the optimal partition is the Voronoi partition $\mathcal{V}(P) = \{V_1, \ldots, V_n\}$ defined by $V_i = \{q \in Q | \; \|q-p_i\| \le \|q-p_j\|, \forall j \ne i\}$. Therefore, an equivalent expression of optimal coverage is $\mathcal{H}(P, \mathcal{V}(P)) = E\left[\max_{i \in \{1,\ldots,n\}} f(\|q-p_i\|)\right]$. Remarkably, one can show [14] that

$$\frac{\partial \mathcal{H}}{\partial p_i}(P, \mathcal{V}(P)) = \int_{V_i \cap B_r(p_i)} \frac{\partial}{\partial p_i} f(\|q-p_i\|) \, d\phi(q),$$

and deduce the following critical property: the gradient of $\mathcal{H}$ is decentralized in the sense that it can be computed with information localized to each individual sphere of influence and Voronoi cell. Closed-form expressions for this partial derivative can be computed under various assumptions on the shape of f.

[Algorithm design] Finally, we design a deplyoment algorithm under the assumption that each agent location obeys a first order dynamical behavior described by $\dot{p}_i = u_i$. Set $u_i = \frac{\partial \mathcal{H}}{\partial p_i}(P, \mathcal{V}(P)) - p_i$, where $\mathcal{V}(P) = \{V_1, \ldots, V_n\}$ is continuously updated in a decentralized computation. This closed-loop system is a gradient flow for the cost function $\mathcal{H}$ so that performance is indeed locally, continuously optimized. The coverage optimization function $\mathcal{H}$ is a Lyapunov function and the group of mobile agents is guaranteed to converge to a local maximum of $\mathcal{H}$. Fig. 1 illustrates the performance of this coordination algorithm when Q is a 2D convex polygon.



initial configuration — decentralized gradient descent — final configuration

Figure 1: Area-coverage deployment for 16 agents; the region of interest is characterized by a density function equal to the sum of Gaussians. The left (resp. right) figure contains the countur plot of the density function, the initial (resp. final) position of the agents, the agents' sphere of influence and Voronoi partitions. The central figure illustrates the joint motion.

P2) Deployment for Maximum Detection Likelihood Next, we consider a second formulation of deployment with a different network objective. We consider n mobile devices equipped with acoustic sensors attempting to detect, identify, and localize a sound-source (we could similarly envision antennas detecting RF signals, or chemical sensors localizing a source). For a variety of criteria, when the source emits a known signal and the noise is Gaussian, we know that (1) the optimal detection algorithm involves a matched filter, (2) detection performance is a function of signal-to-noise-ratio, and, in turn,(3) signal-to-noise ratio is inversely proportional to the sensor-source distance. How do we deploy the agents and optimize their location to maximize the detection probability?

Recall that, the circumcircle of a given polygon is the smallest circle enclosing the polygon; circumradius and circumcenter and radius and center of the circumcircle, respectively. Given this notion, we introduce the following simple algorithm. If each agent moves toward the circumcenter of its Voronoi cell, then, as a function of time, the detection likelihood is inversely proportional to the circumradius of each agent's Voronoi cell, and the detection likelihood is monotonically increasing, see Fig. 2.

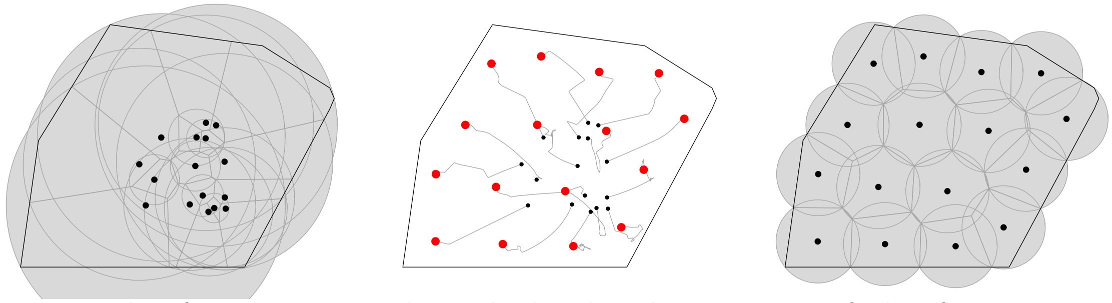


Figure 2: Deployment of 16 agents for maximum likelihood detection. The left (resp. right) figure contains the initial (resp. final) position of the agents, and the Voronoi partitions and circumcircles of each agent.

The fundamental reason this behavior is correct is the existence of an appropriate Lyapunov function, with respect to which the given behavior is dissipative. It turns out that, as a function of the agents' position, an appropriate cost function is the maximum of the radiuses of disks centered at each agent's position and covering each agent Voronoi cell.

P3) Visibility-based Deployment Here we consider a third and final scenario, namely that of deployment of agents in planar non-convex regions. Coverage performance is quantified in terms of visibility: one reasonable coverage objective is to deploy the ad-hoc network in such a way as to obtain complete visibility of the environment. This formulation of the coverage problem is a distributed feedback version of the so-called "art gallery problem." This problem is a classic topic in computational geometry, e.g., see [35, 36]. Let us provide a heuristic algorithm that has demostrated excellent performance in simulations and that is conjectured to have correctness guarantees for sufficiently large numbers of agents. For all $i \in \{1, \dots, n\}$, the ith agent computes its dominance polygon $W_i \subset Q$ as the set of points in the environment for which p_i is either the only visible agent or the closest visible agent. Each agent then moves toward the furthest vertex in its dominance polygon. The outcome is illustrated in Fig. 3.

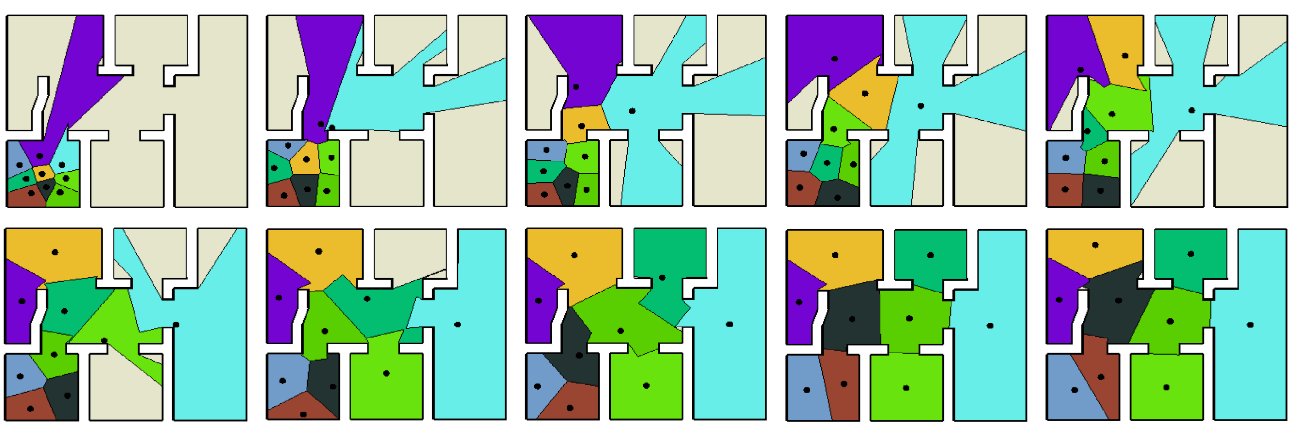


Figure 3: Visibility-based deployment for 9 agents in a somewhat typical floor-plan. Each frame depictes the agents and their respective dominance region.

There are a number of open issues related to this class of visibility-based deployment problems. A first research objective is to provide a correctness proof for this algorithm under the smallest possible assumption on the number of required agents. The extension to the 3 dimensional case would also be very relevant in applications. A final important set of problems is related to the information flow required for the implementation of this algorithm. Note that each agent needs to know the location of every other agent it is sharing some viewpoints with. It is our future objective to weaken this requirement to the simple knowledge of the location of each other visible agent.

3 Scientific Progress and Accomplishments: Experimental advances — the Multi-Rover Network Laboratory

We have developed a multi-vehicle testbed based on all-terrain rovers for indoor and outdoor environments. Each rover is based on an model RC vehicle, a computing and communication suite, and a navigation and sensing suite: all components are commercial off-the-shelf, and most software is open source.

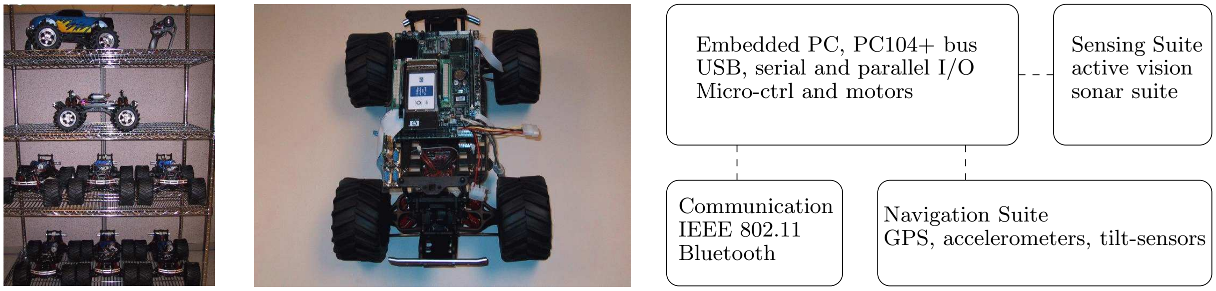


Figure 4: An initial implementation of a mobile autonomous robots based on a model RC truck, a PC104 single board computer, and an 802.11 wireless nic. All components are cots.

(1) The vehicle chassis, motors, wheels, and batteries are those found in the model RC truck "Emaxx" manufactured by the company Traxxas, Inc. For our modified vehicle, maximum speed is about 20mph, expected battery duration is 20minutes, initial payload is 5lb, the model is 20in long and 15in wide, the height off the ground is 10in. The motors are high-voltage high performance, the transmission is all-wheel-drive and two speed. An inexpensive micro-controller from Pontech Inc. provides a serial ASCII-based interface to the motors and speed controller. The model is endowed with a 100 count per revolution optical encoder for odometry.

(2) On-board computation is performed by a single-board embedded computer with PC104-compatible extension bus, 266MHz Pentium processor, disk-on-chip as hard drive, and numerous input-output interfaces. The board is in EBX form factor which is 8in x 5.75in. The board model is VNS-786 manufactured by JUMPTech Adastra Systems Inc. Each processor runs a Mini Real-Time Linux operating system. The implementation is based on the Linux Router project and relies on a tarred compressed file-system.

(3) An extension PC104 board provides the single-board with a PC Card interface. A 802.11b wireless network-interface-card in PC Card format provides local communication. The navigation suite currently includes only a differential-ready GPS receiver, the Garmin GPS 16. The GPS information is transmitted to the embedded PC through a serial line using the standard ASCII protocol NMEA-0183. We plan in the future to purchase tri-axial accelerometers, inclinometers, and a compasses. Additionally, we hope to purchase a suite of sensors, e.g., sonars and cameras.

(4) The experimental platform is being developed in the "Multi-Rover Network Laboratory" in room B16 in the Coordinated Science Laboratory (CSL) at UIUC. The room is about 500ft^2, endowed with currently 3 workstations and 2 development platforms.

(5) The software infrastructure is a multi-threaded priority-based architecture. Threads are being developed to deal with sensors and actuators interfaces, high-level communication protocols, data fusion and estimation algorithms, inner-loop and planning control.

4 List of publications supported by this contract

1. Papers published in peer-reviewed journals

   (a) F. Bullo, "Series expansions for analytic systems linear in controls," *Automatica*, vol. 38, no. 9, pp. 1425–1432, 2002

   (b) W. T. Cerven and F. Bullo, "Constructive controllability algorithms for motion planning and optimization," *IEEE Transactions on Automatic Control*, vol. 48, no. 4, pp. 575–589, 2003

   (c) J. Cortés, S. Martínez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 243–255, 2004

   (d) W. T. Cerven, F. Bullo, and V. L. Coverstone, "Vehicle motion planning with time-varying constraints," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 27, no. 3, pp. 506–508, 2004

2. Papers published in non-peer-reviewed journals or in conference proceedings

   (a) T. Karatas and F. Bullo, "Randomized searches and nonlinear programming in trajectory planning," in *IEEE Conf. on Decision and Control*, (Orlando, FL), pp. 5032–5037, Dec. 2001

   (b) J. Cortés, S. Martínez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," in *IEEE Int. Conf. on Robotics and Automation*, (Arlington, VA), pp. 1327–1332, May 2002

   (c) J. Cortés, S. Martínez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks: variations on a theme," in *Mediterranean Conference on Control and Automation*, (Lisbon, Portugal), July 2002. Electronic Proceedings

   (d) E. Frazzoli and F. Bullo, "On quantization and optimal control of dynamical systems with symmetries," in *IEEE Conf. on Decision and Control*, (Las Vegas, NV), pp. 817–823, Dec. 2002

   (e) C. L. Robinson, D. Block, S. Brennan, F. Bullo, and J. Cortés, "Nonsmooth analysis and sonar-based implementation of distributed coordination algorithms," in *IEEE Int. Conf. on Robotics and Automation*, (New Orleans, LA), pp. 3000–3005, Apr. 2004

   (f) J. Cortés, S. Martínez, and F. Bullo, "Coordinated deployment of mobile sensing networks with limited-range interactions," in *IEEE Conf. on Decision and Control*, (Paradise Island, Bahamas), Dec. 2004. To appear

3. Papers presented at meetings, but not published in conference proceedings

   None.

4. Manuscripts submitted, but not published

   (a) F. Bullo, "Trajectory design for mechanical systems: from geometry to algorithms," *European Journal of Control*, Aug. 2004. To appear

   (b) J. Cortés, S. Martínez, and F. Bullo, "Spatially-distributed coverage optimization and control with limited-range interactions," *ESAIM. Control, Optimisation & Calculus of Variations*, Jan. 2004. Submitted

   (c) S. Aranda, S. Martínez, and F. Bullo, "On optimal sensor placement and motion coordination for target tracking," in *IEEE Int. Conf. on Robotics and Automation*, Sept. 2004. Submitted

5. Technical reports submitted to ARO

   None.

5 Scientific Personnel

1. Francesco Bullo, PI

2. William Todd Cerven, Ph.D., Aeronautical and Astronautical Engineering, UIUC, Jun 2003

3. Sulema Aranda, MS, Electrical and Computer Engineering, UIUC, Aug 2004

4. Timur Karatas, PhD candidate, 5th year, Electrical and Computer Engineering, UIUC

Research Awards and Honors

The following awards were received by the PI during the reporting period:

- SemiPlenary Speaker, International Symposium on Mathematical Theory of Networks and Systems, Leuven, Belgium, Jul 2004
- Plenary Speaker, IFAC Workshop on Lagrangian and Hamiltonian Methods in Nonlinear Control, Sevilla, Spain, Apr 2003
- Xerox Foundation Award for Faculty Research, UIUC College of Engineering, 2003
- Young Investigator Award, Office of Naval Research, 2003
- Best Student Paper Award, IEEE Conf. Decision and Control, Las Vegas NV, Dec 2002
- Invited Speaker, Summer School, Dutch Institute for Systems and Control, Zeist, Netherlands, Jul 2002
- Best Paper Award Finalist, IEEE Conf. Robotics and Automation, Arlington, VA, May 2002

6 Report of Inventions

None.

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[12] J. Cortés, S. Martínez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 243–255, 2004.

[13] J. Cortés and F. Bullo, "Coordination and geometric optimization via distributed dynamical systems," *SIAM Journal on Control and Optimization*, June 2004. To appear.

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